

Weak Mixing Angle and Proton Stability in F-theory GUT

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Abstract

It is pointed out that a class of flipped SU(5) models based on F-theory naturally explains the gauge coupling unification. It is because the group $SU(5) \times U(1)_X$ is embedded in $SO(10)$ and E_8 . To prohibit the dimension 4 and 5 proton decay processes, the structure group should be $SU(3)_\perp$ or smaller. Extra heavy vector-like pairs of $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$ except only one pair of Higgs should be also disallowed, because they could induce the unwanted dimension 5 proton decays. We construct a simple global F-theory model considering these points. To maintain $\sin^2 \theta_W^0 = \frac{3}{8}$ at the GUT scale, the fluxes are turned-on only on the flavor branes.

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I. INTRODUCTION

One of the dramatic successes in the Minimal Supersymmetric Standard Model (MSSM) is the gauge coupling unification. Thanks to the additional contributions by the superpartners to the renormalization effects, the three gauge couplings of the MSSM, $\{g_3, g_2, \sqrt{\frac{5}{3}}g_Y\}$ can be unified quite accurately at 2×10^{16} GeV energy scale [1].¹ It seems to imply the presence of a supersymmetric (SUSY) unified theory at that scale. When discussing the gauge coupling unification in the MSSM, however, one should notice that such a unification is possible, since the normalization for g_Y (and also the normalization for the hypercharges) deduced in SU(5) and SO(10), i.e. $\sqrt{\frac{5}{3}}$ ($\sqrt{\frac{3}{5}}$) is employed. This normalization predicts that the weak mixing angle, which is defined as $\sin^2\theta_W \equiv \frac{g_Y^2}{g_2^2 + g_Y^2}$, should be $\frac{3}{8}$ at the unification scale.

One of the problems in SUSY grand unified theories (GUTs) is the doublet/triplet splitting in the Higgs multiplets. Unlike in the matter sector, the electroweak Higgs in the MSSM, $\{h_u, h_d\}$ can be embedded in proper GUT multiplets [e.g. $\{\mathbf{5}, \bar{\mathbf{5}}\}$ in SU(5) and $\mathbf{10}$ in SO(10)] with unwanted SU(3) triplets $\{D, D^c\}$ supplemented. Although they are contained in a common multiplet, how to make the triplets superheavy while keeping the doublets massless down to the electroweak scale are known to be a notorious problem in GUT.

This problem is closely associated also with the proton decay in SUSY GUTs [2]. While the dimension 4 proton decay processes can be prohibited by introducing the R-parity, the dimension 5 processes can not be forbidden by it. This problem arises often also in the minimal SU(5) and SO(10) in other guises. Even though one successfully splits the doublet/triplets, unless the triplet pieces of the Higgs multiplets are decoupled by an elaborate way, the operators leading to the dimension 5 proton decay are generated again at tree level.

Flipped SU(5), which is based on the gauge group $SU(5) \times U(1)_X$, provides very nice framework addressing these problems [3, 4]. In flipped SU(5), the “missing partner mechanism” for doublet/triplet splitting works in a very simple way [4]. Such split triplets do not induce the dimension 5 proton decay in flipped SU(5). Moreover, in flipped SU(5) there is no serious fermion mass relations constraint by the GUT group structure, which arise often in many simple GUTs. However, the gauge group of flipped SU(5) is a semi-simple group. Thus, it can address the gauge coupling unification, only when it is embedded in a promising UV theory such as string theory; it could determine the $U(1)_Y$ normalization such that $\sin^2\theta_W^0 = \frac{3}{8}$ at the GUT energy scale [5, 6].

In this paper, we attempt to construct a flipped SU(5) model based on F-theory. We will point out that the predicted $\sin^2\theta_W^0$ at the string scale, which is assumed to be around the

¹ For the hypercharges of the MSSM superfields, we take the convention of $Y[q] = \frac{1}{6}$, $Y[u^c] = -\frac{2}{3}$, $Y[l] = -\frac{1}{2}$, etc. throughout this paper. In our notation, q , $u^c(d^c)$, l , and $e^c(\nu^c)$ mean the quark doublet, quark singlet with $Q_{\text{em}} = -\frac{2}{3}(+\frac{1}{3})$, lepton doublet, and lepton singlet with $Q_{\text{em}} = +1(0)$, respectively. For the superheavy fields carrying the same quantum numbers with the MSSM fields, mainly the capital letters will be utilized in this paper.

GUT scale, is $\frac{3}{8}$.² Hence the three gauge couplings in the MSSM [or SU(5) and U(1)_X gauge couplings] are unified at the GUT scale. In order to obtain the chiral fields in 4 dimensional spacetime (4D) and to maintain the gauge coupling unification, we will turn on the universal fluxes only on the flavor branes. We will also discuss how to forbid dimension 4 and 5 proton decay processes in the flipped SU(5) model based on F-theory such that the dimension 6 process [$p \rightarrow e^+ \pi^0$ with $\tau_p \approx 10^{34-35}$ yr] becomes the dominant one.

F-theory is defined by lifting the $SL(2, \mathbb{Z})$ symmetry of Type IIB string theory to that of geometric torus. The axion-dilaton field in IIB string is identified to the complex structure of the torus [8, 9]. Toward a four dimensional $N = 1$ SUSY model, we compactify F-theory on Calabi-Yau fourfold, which is elliptically fibred on a three-base B .

The varying axion-dilaton field on B , which naturally incorporates non-perturbative effects, makes more light degrees of freedom than open fundamental strings possible, so that exceptional group of E_n series emerges. Identifying E_3, E_4, E_5 as SU(3)×SU(2), SU(5) and SO(10), respectively, we have natural symmetry enhancement patterns $E_3 \times U(1)_Y \times U(1)_X \subset E_4 \times U(1)_X \subset E_5$ [10–12]. Thus, F-theory enables us to track how such unification pattern is realized.

In particular $E_4 \times U(1)_X$ naturally provides the flipped SU(5) group: not only gauge group but also matter contents and Yukawa couplings nicely fit [11, 14, 15]. As mentioned above, we will try to construct an F-theory model to reproduce a field-theoretically desired flipped SU(5) model, particularly addressing the issues on the gauge coupling unification and the absence of dimension 4 and 5 proton decay operators [11, 16]. Conventional GUTs employ Higgs scalar fields to break GUT groups to the SM group. The Higgs mechanism in SUSY GUTs could inherit the gauge coupling unification of the MSSM. In F-theory GUT, there is another way of GUT breaking using flux. A flux along the hypercharge direction, however, is known to distort a little bit the gauge coupling unification [13, 17]. To track the origin of the observed value of $\sin^2 \theta_W$, we will consider an F-theory GUT as not E_4 but $E_4 \times U(1)$, whose breaking solely relies on the Higgs mechanism.

This paper is organized as follows. In section II, we will briefly review flipped SU(5) and discuss dimension 4 and 5 proton decay in flipped SU(5). In section III, we construct an F-theory model of flipped SU(5). When constructing a model, we will particularly focus on how to reflect the gauge coupling unification observed in the MSSM, and to avoid the dimension 4 and 5 proton decay processes. In section IV, we will discuss low energy physics expected from our F-theory model. Section V will be devoted to conclusions.

² In the strongly coupled heterotic string theory (or heterotic M-theory), the fundamental scale becomes coincident with the GUT scale [7]. As dual to the heterotic M-theory, F-theory has the same relation.

$SU(5)_X$	MSSM fields
$\mathbf{10}_1$	$\{d^c, q, \nu^c\}$
$\bar{\mathbf{5}}_{-3}$	$\{u^c, l\}$
$\mathbf{1}_5$	e^c
$\mathbf{5}_{-2} (\equiv \mathbf{5}_h)$	$\{D, h_d\}$
$\bar{\mathbf{5}}_2 (\equiv \bar{\mathbf{5}}_h)$	$\{D^c, h_u\}$

TABLE I: Superfields in flipped SU(5). The SU(3) triplets D and D^c are absent in the MSSM, which should be decoupled from low physics. When flipped SU(5) embedded in SO(10), the X charges in the table should be normalized as $X \rightarrow \frac{1}{\sqrt{40}}X$.

II. FLIPPED SU(5)

The gauge group of flipped SU(5) is $SU(5) \times U(1)_X$. Unlike in the conventional SU(5) i.e. Georgi-Glashow's SU(5) [$\equiv SU(5)_{\text{GG}}$] [18], the hypercharge of the standard model (SM) is defined as a linear combination of a diagonal SU(5) and $U(1)_X$ generators:

$$Y = \frac{1}{5} (T_5 + X) , \quad (1)$$

where $T_5 [= \text{diag.}(\frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{-1}{2} \frac{-1}{2})]$ is a diagonal generator of SU(5), and X denotes the $U(1)_X$ charge. For a while, let us neglect the normalizations of T_5 and X . Table I lists the field contents of the flipped SU(5) model, from which one can see how the MSSM superfields are embedded there.

Breaking of $SU(5) \times U(1)_X$ to the SM gauge group demands introduction of the Higgs fields $\{\mathbf{10}_H, \bar{\mathbf{10}}_H\}$, which carry the quantum numbers of $\{\mathbf{10}_1, \bar{\mathbf{10}}_{-1}\}$, respectively. Since they contain the SM singlets $\{\nu_H^c, \bar{\nu}_H^c\}$, their vacuum expectation values (VEVs) in the SM singlet directions result in spontaneous breaking of flipped SU(5) to the SM gauge group.

The different definition of the hypercharge results in the different embedding of the MSSM fields: comparing with $SU(5)_{\text{GG}}$, d^c , ν^c and h_d are replaced by u^c , e^c , and h_u , respectively. As a result, the prediction from Yukawa couplings in flipped SU(5) is also different from that of the conventional SU(5). The superpotential in flipped SU(5) is written down as follows:

$$W = y_{ij}^{(d)} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_h + y_{ij}^{(u,\nu)} \mathbf{10}_i \bar{\mathbf{5}}_j \bar{\mathbf{5}}_h + y_{ij}^{(e)} \mathbf{1}_i \bar{\mathbf{5}}_j \mathbf{5}_h + \mu \mathbf{5}_h \bar{\mathbf{5}}_h + \frac{y_{ij}^{(m)}}{M_P} \bar{\mathbf{10}}_H \bar{\mathbf{10}}_H \mathbf{10}_i \mathbf{10}_j , \quad (2)$$

where i, j stand for the family indices. From the first term, d-type quarks [rather than u-type quarks as in the $SU(5)_{\text{GG}}$] get masses. From the second term, u-type quarks' and Dirac neutrinos' masses are generated, and they are related as $M_{ij}^{(u)} = M_{ji}^{(\nu)}$ [rather than $M_{ij}^{(d)} = M_{ji}^{(e)}$]. However this relation is not much crucial, because the physical neutrino masses are given by the Majorana mass terms as well as the Dirac mass terms. The Majorana masses are induced by the last term of Eq. (2), when $\bar{\mathbf{10}}_H$ develop a VEV in the right-handed neutrino direction. Thus, there is no effective mass relation in flipped SU(5), and so

unrealistic mass relations predicted in other simple GUT models are absent. The charged leptons achieve the masses from the third term of Eq. (2). From now on, we will provide some comments on flipped SU(5) in order.

A. Weak Mixing Angle and Coupling Unification

Normalization of U(1) charges seems arbitrary, since rescaling of the charges can be absorbed by the coupling constant. The same can be true for X of bottom-up constructed flipped SU(5). But this is not the case if U(1) $_X$ is embedded in a simple group, since then U(1) coupling becomes not independent. If SU(5) \times U(1) $_X$ is embedded in SO(10), it should be fixed to $\frac{1}{\sqrt{40}}X$. In such a case, hence, $\mathbf{10}_1$, $\mathbf{5}_{-3}$, $\mathbf{1}_5$, etc. in Table I should be replaced by $\mathbf{10}_{1/\sqrt{40}}$, $\mathbf{5}_{-3/\sqrt{40}}$, $\mathbf{1}_{5/\sqrt{40}}$, and so forth. Even in such a case, however, we will drop the normalization factor in the subscripts, just tacitly assuming it for simplicity in notations. Indeed, the U(1) $_X$ charge normalization by $\frac{1}{\sqrt{40}}$ yields $\sin^2\theta_W^0 = \frac{3}{8}$, unifying the SU(5) and U(1) $_X$ gauge couplings at the GUT scale (see e.g. appendix of Ref. [6]).

As mentioned in Introduction, there are many difficult problems such as the doublet/triplet splitting problem of the Higgs sector in ordinary 4 dimensional SUSY GUTs. Hence, it would be desirable to construct a flipped SU(5) model in the framework of string theory such that the normalization of the U(1) $_X$ charges is given by $\frac{1}{\sqrt{40}}$ [5, 6]. In that case, the flipped SU(5) gauge group is embedded in a much larger group, but it is broken to SU(5) \times U(1) $_X$ not by a spontaneous breaking mechanism but by a way associated with a compactification mechanism of the extra space dimensions. Such an explicit construction of flipped SU(5) from string theory with realizing the desired normalization of U(1) $_X$ could easily avoid the problems appearing in SUSY GUTs.

B. Missing Partner Mechanism

Flipped SU(5) can be broken to the SM gauge group by the tensor Higgs $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$ carrying the X charges +1 and -1, respectively. In terms of the SM quantum numbers, the tensor Higgses $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$ split to $\{d_H^c, q_H, \nu_H^c\}$ and $\{d_H, \bar{q}_H, \nu_H\}$, respectively. When $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$ develop VEVs along the ν_H^c and ν_H directions, q_H and \bar{q}_H are absorbed by the heavy gauge sector, but d_H^c and d_H contained in $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$ potentially remain as pseudo Goldstone modes. Somehow they should be made superheavy to protect the gauge coupling unification.

$\{D, D^c\}$ modes included in $\{\mathbf{5}_h, \bar{\mathbf{5}}_h\}$ should be also removed from the low energy field spectrum, while the doublets in $\{\mathbf{5}_h, \bar{\mathbf{5}}_h\}$ should survive down to low energies because they are nothing but the electroweak Higgs in the MSSM. This is the doublet/triplet splitting problem in flipped SU(5). However, the unwanted $\{d_H^c, d_H\}$ from $\{\mathbf{10}_H, \overline{\mathbf{10}}_H\}$ and $\{D, D^c\}$ from $\{\mathbf{5}_h, \bar{\mathbf{5}}_h\}$ turn out to be superheavy by pairing with each other. It is a merit of flipped

SU(5). Consider the following superpotential,

$$W \supset \mathbf{10}_H \mathbf{10}_H \mathbf{5}_h + \overline{\mathbf{10}}_H \overline{\mathbf{10}}_H \overline{\mathbf{5}}_h = \langle \nu_H^c \rangle d_H^c D + \langle \bar{\nu}_H^c \rangle d_H D^c , \quad (3)$$

which is allowed in flipped SU(5). As seen in Eq. (3), all the unwanted modes discussed above become superheavy by obtaining the Dirac masses proportional to $\langle \mathbf{10}_H \rangle$ and $\langle \overline{\mathbf{10}}_H \rangle$. However, one should note that this mechanism works for only one pair of vector-like Higgs fields. If there are more heavy Higgs-like fields $\{\mathbf{5}_G, \overline{\mathbf{5}}_G\}$, the triplet modes included there can not get masses through this mechanism; introducing another pairs $\{\mathbf{10}'_H, \overline{\mathbf{10}}'_H\}$ for removing such triplets would leave unwanted pseudo Goldstones $\{q'_H, \bar{q}'_H\}$ contained in $\{\mathbf{10}'_H, \overline{\mathbf{10}}'_H\}$, which can not eaten by the gauge sector.

C. Proton Stability

In the MSSM the baryon and lepton numbers are conserved by R-parity at the renormalizable level. (It might be an ad hoc introduction for the baryon and lepton number conservation, and dark matter.) Even R-parity, however, can not prohibit the dimension 5 proton decay processes. In flipped SU(5), the R-parity violating terms in the MSSM do not arise from the renormalizable superpotential at all, because they are forbidden by U(1)_X [unlike in SU(5)_{GG}]. However, such R-parity violating terms as well as the terms leading to dimension 5 proton decay can appear from the non-renormalizable superpotential:

$$\frac{1}{M_P} \mathbf{10}_H \mathbf{10}_i \mathbf{10}_j \overline{\mathbf{5}}_k \rightarrow \frac{\langle \nu_H^c \rangle}{M_P} (q_i d_j^c l_k + d_i^c d_j^c u_k^c) , \quad \frac{1}{M_P} \mathbf{10}_H \overline{\mathbf{5}}_i \overline{\mathbf{5}}_j \mathbf{1}_k \rightarrow \frac{\langle \nu_H^c \rangle}{M_P} l_i l_j e_k^c , \quad (4)$$

$$\frac{1}{M_P} \mathbf{10}_i \mathbf{10}_j \mathbf{10}_k \overline{\mathbf{5}}_l \rightarrow \frac{1}{M_P} q_i q_j q_k l_l , \quad \frac{1}{M_P} \mathbf{10}_i \overline{\mathbf{5}}_j \overline{\mathbf{5}}_k \mathbf{1}_l \rightarrow \frac{1}{M_P} d_i^c u_j^c u_k^c e_l^c , \quad (5)$$

where i, j, k, l indicate again the family indices. These terms in the superpotential should be forbidden somehow for the baryon and lepton number conservations. Then, proton decay would be dominated by dimension 6 operators, which are still safe for the proton longevity. But it is not the end of the discussion.

Let us suppose that there is an extra vector-like pair of $\{\mathbf{5}_G, \overline{\mathbf{5}}_G\}$, which carries the same quantum numbers with the electroweak Higgs pair $\{\mathbf{5}_h, \overline{\mathbf{5}}_h\}$. Then the allowed superpotential is as follows:

$$W_{\text{unwanted}} = \mathbf{10}_i \mathbf{10}_j \mathbf{5}_G + \mathbf{10}_k \overline{\mathbf{5}}_i \overline{\mathbf{5}}_G + \mathbf{1}_m \overline{\mathbf{5}}_n \mathbf{5}_G + M_G \mathbf{5}_G \overline{\mathbf{5}}_G , \quad (6)$$

where M_G is supposed to be a GUT or Planck scale mass parameter. Hence, the extra pair $\{\mathbf{5}_G, \overline{\mathbf{5}}_G\}$ achieves a superheavy Dirac Mass M_G . In terms of the SM, $\mathbf{5}_G$ and $\overline{\mathbf{5}}_G$ split into $\{D_G, L_G\}$ and $\{D_G^c, L_G^c\}$, respectively. The first three terms of Eq. (6) are presented as

$$(d_{ij}^c \nu_j^c + q_i q_j + e_m^c u_n^c) D_G + (d_{ij}^c q_j + e_m^c l_n) L_G + (d_k^c u_l^c + q_k l_l) D_G^c + (q_k u_l^c + \nu_k^c l_l) L_G^c , \quad (7)$$

Note that after integrating out the heavy $\{D_G, D_G^c\}$ modes included in $\{\mathbf{5}_G, \bar{\mathbf{5}}_G\}$, the unwanted terms of Eq. (5) are generated again. They are suppressed by $1/M_G$ (rather than $1/M_P$). Thus, the extra pair of $\{\mathbf{5}_G, \bar{\mathbf{5}}_G\}$ are also dangerous for proton stability, even if they are superheavy. In the case of the SM gauge symmetry, this problem could arise also, if there are extra vector-like pairs of heavy SU(3) triplets.

On the other hand, $\{D, D^c\}$ included in the Higgs multiplets $\{\mathbf{5}_h, \bar{\mathbf{5}}_h\}$ become superheavy by pairing with $\{d_H^c, d_H\}$ contained in $\{\mathbf{10}_H, \bar{\mathbf{10}}_H\}$ as discussed in subsection B, and the mass parameter corresponding to M_G of Eq. (6), namely, “ μ ” in Eq. (2) is just of the electroweak scale. Accordingly, the terms induced by $\{D, D^c\}$ are suppressed by $\mu/\langle\mathbf{10}_H\rangle^2$ rather than $1/M_P$, which are extremely small.

III. CONSTRUCTION FROM F-THEORY

Before constructing a model from F-theory, let us discuss first some results inferred by considering only the gauge invariance and the notion of monodromy. The low energy theory would be eventually embedded in E_8 : all the SM matter originate from the branching of its gaugino. Namely, under $E_8 \rightarrow \text{SU}(5) \times \text{U}(1)_X \times \text{SU}(4)_\perp$, the adjoint branches as

$$\begin{aligned} 248 \rightarrow & (\mathbf{24}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{15})_0 + (\mathbf{1}, \mathbf{1})_0 \\ & + [(\mathbf{1}, \mathbf{4})_5 + (\mathbf{5}, \bar{\mathbf{6}})_{-2} + (\bar{\mathbf{5}}, \mathbf{4})_{-3} + (\mathbf{10}, \mathbf{4})_1 + (\mathbf{10}, \mathbf{1})_{-4} + \text{c.c.}] . \end{aligned} \quad (8)$$

Focusing on $\text{SU}(5) \times \text{U}(1)_X$ quantum numbers, we see it reproduces the desired matter contents in the minimal way. The only unwanted one is the only $\text{SU}(4)_\perp$ singlet $(\mathbf{10}, \mathbf{1})_{-4}$. We can easily remove it from low energy field spectrum just by manipulating G -flux. We will discuss it again later.

We achieve the desired symmetry breaking by embedding a background gauge bundle of the structure group $\text{SU}(4)_\perp \times \text{U}(1)_X$. The unbroken group is the commutant group in E_8 , i.e. the flipped SU(5) group, $\text{SU}(5) \times \text{U}(1)_X$. Note that $\text{U}(1)_X$ can be unbroken, because it commutes with itself. The issue concerning its anomaly will be discussed later. The important properties of the gauge bundle are the followings. First, the zero mode solution under this background becomes chiral: the undisplayed complex conjugate, “c.c.” corresponding to each displayed matter in Eq. (8) appears as just an anti-particle state to form chiral matter [19–22]. Second, considering an instanton background in heterotic dual theory, the actual physical degree is only that modded out by S_4 monodromy. It can be realized by a “spectral cover.”

A. Monodromy

To study its consequence, it is convenient to deal with the weights of $\mathbf{4}$ as $\{t_1, t_2, t_3, t_4\}$. Our S_4 is the permutation group shuffling all of these four weights. We can also associate

$U(1)_X$ charged $SU(4)_\perp$ singlet $\mathbf{1}_X$ as $\{t_5\}$. It is understood as embedding $SU(4)_\perp \times U(1)_X \subset SU(5)_\perp$, under which $\mathbf{4} + \mathbf{1}_X \rightarrow \mathbf{5}$.

The $U(1)_X$ quantum numbers subscripted in Eq. (8) are correctly reproduced by assigning

$$X = (1, 1, 1, 1; -4) \quad (9)$$

in the $(t_1, t_2, t_3, t_4, t_5)$ basis. Modding out by S_4 , we have two kinds of $\mathbf{10}$ representations

$$\mathbf{10}_i : \{t_1, t_2, t_3, t_4\} , \quad \text{and} \quad \mathbf{10}_{-4} : \{t_5\} .$$

Likewise, S_4 distinguishes two kinds of $\mathbf{5}$'s,

$$\begin{aligned} \bar{\mathbf{5}}_h &: \{t_1 + t_2, t_1 + t_3, t_1 + t_4, t_2 + t_3, t_2 + t_4, t_3 + t_4\} , \\ \bar{\mathbf{5}}_i &: \{t_1 + t_5, t_2 + t_5, t_3 + t_5, t_4 + t_5\} . \end{aligned}$$

In the same way, we can identify the $SU(5)$ singlets. We see that the matter fields naturally compose the $SO(10)$ multiplets.

The Yukawa couplings of Eq. (2) are deduced from the gauge invariant Chern–Simons interactions [10], having the structure

$$\begin{aligned} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_h &: (t_m) + (t_n) + (-t_m - t_n) = 0 , \\ \mathbf{10}_i \bar{\mathbf{5}}_j \bar{\mathbf{5}}_h &: (t_m) + (t_n + t_5) + (t_p + t_q) = 0 , \\ \mathbf{1}_i \bar{\mathbf{5}}_j \mathbf{5}_h &: (t_m - t_5) + (t_n + t_5) + (-t_m - t_n) = 0 , \\ \bar{\mathbf{10}}_H \bar{\mathbf{10}}_H \mathbf{10}_i \mathbf{10}_j &: (-t_m) + (-t_n) + (t_m) + (t_n) = 0 , \end{aligned} \quad (10)$$

where all the indices, m, n, p, q run over 1, 2, 3, 4 and are different. Later we will distinguish $\mathbf{10}_i$ and $\mathbf{10}_H$ only by the vacuum expectation value (VEV): the $\mathbf{10}$ developing a nonzero VEV is regarded as $\mathbf{10}_H$. We cannot distinguish them by introducing another monodromy, since it is simply a vector representation under $U(4)_\perp$. To justify the second line of Eq. (10), we have to impose the traceless relation,

$$t_1 + t_2 + t_3 + t_4 + t_5 = 0 . \quad (11)$$

Hence, the ‘trace part’ of $U(4)$ is cancelled by $U(1)_X$. Thus, sometimes the structure group is suggestively denoted by $S[U(4)_\perp \times U(1)_X]$. However, we also find that there are couplings like

$$\mathbf{10}_1 \mathbf{10}_1 \mathbf{10}_1 \bar{\mathbf{5}}_{-3} : (t_i) + (t_j) + (t_k) + (t_l + t_5) = 0 , \quad (12)$$

yielding proton decay operators, $d^c d^c u^c$ in Eq. (4) and $qqql$ in Eq. (5) at tree level.

The best remedy is to further decompose $S[U(4)_\perp \times U(1)_X] \rightarrow S[U(3)_\perp \times U(1)_Z \times U(1)_X]$ by singling out t_4 , and introduce S_3 monodromy on the $U(3)$ part. Observing the quantum number, it is easy to find the spectrum, summarized in Table II. There is a new commutant group $U(1)_Z$, generated by

$$Z = (1, 1, 1, -3, 0) \quad (13)$$

Matter	Matter Curve	Homology Class	Net # of Families
$\mathbf{10}_1$	$\prod_i t_i \rightarrow 0$	$\sigma \cap (\eta - 3c_1)$	$-(\lambda\eta - \frac{1}{3}\zeta) \cdot (\eta - 3c_1) = 3$
$\mathbf{10}'_1$	$t_4 \rightarrow 0$	$\sigma \cap (-c_1)$	$c_1 \cdot \zeta = 0$
$\bar{\mathbf{5}}_{-3}$	$\prod_i (t_i + t_5) \rightarrow 0$	$\sigma \cap (\eta - 3c_1)$	$-(\lambda\eta - \frac{1}{3}\zeta) \cdot (\eta - 3c_1) = 3$
$\bar{\mathbf{5}}'_{-3}$	$t_4 + t_5 \rightarrow 0$	$\sigma \cap (-c_1)$	$c_1 \cdot \zeta = 0$
$\mathbf{1}_5$	$\prod_i (t_i - t_5) \rightarrow 0$	$\sigma \cap (\eta - 3c_1)$	$-(\lambda\eta - \frac{1}{3}\zeta) \cdot (\eta - 3c_1) = 3$
$\mathbf{1}'_5$	$t_4 - t_5 \rightarrow 0$	$\sigma \cap (-c_1)$	$c_1 \cdot \zeta = 0$
$\mathbf{5}_{-2} (\equiv \mathbf{5}_h)$	$\prod_{i,j} (-t_i - t_j) \rightarrow 0$	$(2\sigma + \eta) \cap (\eta - 3c_1)$	$-(\lambda\eta + \frac{2}{3}\zeta) \cdot (\eta - 3c_1) = 1$
$\bar{\mathbf{5}}'_2 (\equiv \bar{\mathbf{5}}_h)$	$\prod_i (t_i + t_4) \rightarrow 0$	$\sigma \cap (\eta - 3c_1)$	$-(\lambda\eta + \frac{2}{3}\zeta) \cdot (\eta - 3c_1) = 1$
$\mathbf{1}_0$	$\prod_i (t_i - t_4) \rightarrow 0$	$\sigma \cap (\eta - 3c_1)$	$-(\lambda\eta - \frac{4}{3}\zeta) \cdot (\eta - 3c_1) = 5$
$\mathbf{10}_{-4}$	$t_5 \rightarrow 0$	$\sigma \cap (-c_1)$	0

TABLE II: Field spectrum in the F-theory model of flipped SU(5). Fluxes $\lambda(3\sigma_\infty - \eta) + \frac{1}{3}\zeta$ and $-\zeta$ are turned-on on $C^{(a)}$ and $C^{(b)}$, respectively. We take $\lambda = \frac{1}{6}$, $\eta \cdot (\eta - 3c_1) = -14$, $\eta \cdot \zeta = 2$, and $c_1 \cdot \zeta = 0$ for obtaining three families of matter and only one pair of the electroweak Higgs.

in the same basis. Since $\mathbf{10}_1$ representation can take only one of weights t_1, t_2, t_3 *except* t_4 and we assign $\bar{\mathbf{5}}_{-3}$ matter as $\{t_i + t_5, i = 1, 2, 3\}$, it is impossible to satisfy (12). If there is no $\bar{\mathbf{5}}'_{-3} : \{t_4 + t_5\}$ due to G -flux, shown in Table II, we have no dangerous dimension 4 operators of Eqs. (4) and the dimension 5 operators of Eq. (5).

B. Matter Curves

To have four dimensional $N = 1$ SUSY, we compactify F-theory on an elliptic Calabi–Yau fourfold. Our $\text{SU}(5) \times \text{U}(1)_X$ gauge group is located at a codimension 1 complex surface S_{GUT} in the base B of the elliptic fiber. In analogy to perturbative Type IIB string, we interpret that a stack of sevenbranes wraps S_{GUT} and the rest of the direction to be our 4 noncompact spacetime dimensions.

In this subsection, only the structure group will be described, and the concrete realization of S_{GUT} will be given in the following subsection. To obtain the transformation property reflecting monodromy, we introduce a spectral cover [23]. It encodes the symmetry breaking information. The information on the structure group $\text{S}[\text{U}(3) \times \text{U}(1)_Z \times \text{U}(1)_X]$ is contained in the spectral covers $C^{(a)} \cup C^{(b)} \cup C^{(d)}$. It is described by the algebraic equation,

$$P_a P_b P_c \equiv (a_0 U^3 + a_1 U^2 V + a_2 U V^2 + a_3 V^3)(b_0 U + b_1 V)(d_0 U + d_1 V) = 0, \quad (14)$$

where each factor corresponds to the cover with the same index. Here we consider a conventional dual space to B via projectivization:

$$\check{Z} = \mathbb{P}(K_S \oplus \mathcal{O}) \xrightarrow{\pi} S_{\text{GUT}},$$

where K_S and \mathcal{O} indicate the canonical and trivial bundles on S_{GUT} , respectively. U and V parameterize respectively the zero section σ and the section at infinity $\sigma_\infty \equiv \sigma + \pi^*c_1(S_{\text{GUT}})$ such that $\sigma \cap \sigma_\infty = 0$ (see e.g. Ref. [24]). In other words, $U = 0$ is the location of S_{GUT} . On S_{GUT} , hence, a_m are sections of $-t + (6 - m)c_1$, where c_1 and $-t$ symbolize the first Chern classes of the tangent bundle of S_{GUT} and the normal bundle to S_{GUT} in B . Also both b_1/b_0 and c_1/c_0 transform as $-c_1$.

We can relate weights $\mathbf{3} : \{t_1, t_2, t_3\}$, $\mathbf{1}_Z : \{t_4\}$, $\mathbf{1}_X : \{t_5\}$ of the structure group and the positions of the spectral covers as

$$\begin{aligned} a_1/a_0 &\sim t_1 + t_2 + t_3 , \\ a_2/a_0 &\sim t_1t_2 + t_1t_3 + t_2t_3 , \\ a_3/a_0 &\sim t_1t_2t_3 , \\ b_1/b_0 &\sim t_4 , \\ d_1/d_0 &\sim t_5 , \end{aligned} \tag{15}$$

reflecting the S_3 monodromy. The unimodular condition Eq. (11) implies $a_0b_0d_1 + a_0b_1d_0 + a_1b_0d_0 = 0$, with which the three covers can not be independent. To be consistent with the Green-Schwarz relation in 6 dimensions, b_0 and d_0 should be the trivial sections on S_{GUT} [12, 25, 26]. So we set

$$b_0 = d_0 = 1 . \tag{16}$$

Thus, the traceless condition of $\text{SU}(5)_\perp$ becomes

$$a_1 = -a_0(b_1 + d_1) . \tag{17}$$

The matter field appears at a curve, along which the gauge symmetry is enhanced [21]. As discussed before, the off-diagonal components from the branching yield chiral matter. In \check{Z} , a certain factor of the spectral cover (and combinations thereof) intersect S_{GUT} along such matter curves. From the weight vectors, as presented in Table II, one can see which combinations of the spectral covers give the specific matter fields. For instance, $\mathbf{10}_1$ matter field associated with $t_1t_2t_3 \rightarrow 0$, is localized at the curve $\{a_3 = 0\}$. It is obtained from

$$C^{(a)} \cap \sigma = \pi_a^*(\eta - 3c_1) \cap \sigma . \tag{18}$$

Setting $U = 0$ in the equation for $C^{(a)}$, we indeed obtain the equation $a_3 = 0$ on S_{GUT} . Note that V can not be zero when $U = 0$.

The Higgs field $\mathbf{5}_{-2}$ appears as $\prod_{1 \leq i, j \leq 3} (t_i + t_j) \rightarrow 0$. Since both t_i and t_j are inside $C^{(a)}$, we expect that the corresponding curve comes from the intersection $C^{(a)} \cap \tau C^{(a)}$. They are the common solutions of $P_a(V) = 0$ and $P_a(-V) = 0$, or

$$U(a_0U^2 + a_2V^2) = 0 \quad \text{and} \quad V(a_1U^2 + a_3V^2) = 0 .$$

Since $C^{(a)} \cap \sigma$ or $U = a_3 = 0$ correspond to $\mathbf{10}_1$, which has been already counted, now we don't consider this possibility for $\mathbf{5}_{-2}$. Also another redundant solution is $V = a_0 = 0$. We also drop it since $V = 0$ is infinitely far from S_{GUT} . Thus, the remaining equation we should solve for $C^{(a)} \cap \tau C^{(a)}$ is $a_0 U^2 + a_2 V^2 = a_1 U^2 + a_3 V^2 = 0$. However, still we have an irrelevant solution, $V^2 = a_0 = 0$. Here it should be noted that $a_1 = 0$ by Eq. (17) if $a_0 = 0$. Hence we should drop it also. Then the remaining solution, which corresponds to $\mathbf{5}_{-2}$, becomes associated with the following homology class;

$$\bar{\mathbf{5}}_2 : (\pi^* \eta + 2\sigma) \cap \{\pi^*(\eta - c_1) + 2\sigma\} - 2\sigma_\infty \cap \pi^* \eta = (2\sigma + \pi^* \eta) \cap \pi^*(\eta - 3c_1). \quad (19)$$

Here we used $\sigma \cap \sigma = -c_1 \cap \sigma$.

Similarly, by surveying the index structure one can see where the other matter curves are located. For the curves inside $C^{(i)} \cap \tau C^{(j)}$, $i, j = a, b, d$, we look for the common solution of $P_i(V) = 0$ and $P_j(-V) = 0$, drop the redundant part, and read off the homology. The results are

$$\mathbf{10}'_1 : \sigma \cap \pi^*(-c_1) \in C^{(b)} \cap \tau C^{(b)}, \quad (20)$$

$$\mathbf{10}_{-4} : \sigma \cap \pi^*(-c_1) \in C^{(d)} \cap \tau C^{(d)}, \quad (21)$$

$$\bar{\mathbf{5}}'_2 : \sigma \cap \pi^*(\eta - 3c_1) \in C^{(a)} \cap \tau C^{(b)}, \quad (22)$$

$$\bar{\mathbf{5}}_{-3} : \sigma \cap \pi^*(\eta - 3c_1) \in C^{(a)} \cap \tau C^{(d)}, \quad (23)$$

$$\bar{\mathbf{5}}'_{-3} : \sigma \cap \pi^*(-c_1) \in C^{(b)} \cap \tau C^{(d)}. \quad (24)$$

For the curves inside $C^{(i)} \cap C^{(j)}$, we also find the common solutions of $P_i(V) = 0$ and $P_j(V) = 0$, but this is meaningful only for $i \neq j$. The results are

$$\mathbf{1}_0 : \sigma \cap \pi^*(\eta - 3c_1) \in C^{(a)} \cap C^{(b)}, \quad (25)$$

$$\mathbf{1}_5 : \sigma \cap \pi^*(\eta - 3c_1) \in C^{(a)} \cap C^{(d)}, \quad (26)$$

$$\mathbf{1}'_5 : \sigma \cap \pi^*(-c_1) \in C^{(b)} \cap C^{(d)}. \quad (27)$$

The number of generations in 4 dimension presented in the last column in Table II is determined after fluxes are turned-on. We will discuss it in subsection D.

C. Elliptic Equation

By definition of F-theory, the Calabi–Yau fourfold contains a torus, which is described by the elliptic equation,

$$y^2 + \mathbf{a}_1 xy + \mathbf{a}_3 y = x^3 + \mathbf{a}_2 x^2 + \mathbf{a}_4 x + \mathbf{a}_6. \quad (28)$$

The coefficients \mathbf{a}_m are the sections of $(-m)$ -th power of the canonical bundle K_B for the vanishing first Chern class of the Calabi–Yau fourfold. By reading off the dependence of

\mathbf{a}_m 's on the normal direction to S_{SUT} , we can identify the gauge group on S_{GUT} . Tate's classification for the simple groups is tabulated e.g. in Ref. [27]. For a semi-simple (possibly plus Abelian) group, we can construct a similar equation using the information on the spectral cover [28]. Expanding the spectral cover Eq. (14), we have the special combinations of a_m, b_m and d_m as the coefficients of $U^k V^{5-k}$. These combinations enter as the coefficients of the elliptic equation Eq. (28):

$$\begin{aligned} \mathbf{a}_1 &= -a_3 b_1 d_1 + O(z), \\ \mathbf{a}_2 &= (a_2 b_1 d_1 + a_3 b_1 + a_3 d_1)z + O(z^2), \\ \mathbf{a}_3 &= -(a_1 b_1 d_1 + a_2 b_1 + a_2 d_1 + a_3)z^2 + O(z^2), \\ \mathbf{a}_4 &= (a_0 b_1 d_1 + a_1 b_1 + a_1 d_1 + a_2)z^3 + O(z^4), \\ \mathbf{a}_6 &= a_0 z^5 + O(z^6), \end{aligned} \tag{29}$$

where z parameterizes the normal space to S_{GUT} in B . The other parameters in Eq. (29) are those appearing in Eq. (14), the spectral cover for the $\text{SU}(5) \times \text{U}(1)_X \times \text{U}(1)_Z$ group. For the 'unfactorized' $\text{SU}(5) \times \text{U}(1)_X$ case, one can obtain the corresponding equation in a similar way.

Completing the square in y on the left-hand-side of Eq. (28), the discriminant of the remainder in x takes the following form;

$$\begin{aligned} \Delta &= a_3^4 b_1^4 d_1^4 (b_1 + d_1)(a_3 + a_2 b_1 + a_2 d_1)(a_3 + a_2 b_1 - a_0 b_1^2 - a_0 b_1^2 d_1)(a_3 + a_2 d_1 - a_0 b_1 d_1^2) z^5 \\ &\quad + a_3^2 b_1^2 d_1^2 P z^6 + Q z^7 + O(z^8), \end{aligned} \tag{30}$$

where we used the traceless constraint Eq. (17) to eliminate a_1 and the coefficients P and Q are not proportional to a_3, b_1 and d_1 . The coefficient of z^5 is factorized to give various matter curve equations on S_{GUT} , obtained in section IIIB. One obvious limit is $d_1 \rightarrow 0$, in which the gauge symmetry is enhanced to $O(z^7)$, which yields $\text{SO}(10)$. Other limits such as $a_3 \rightarrow 0$ or $b_1 \rightarrow 0$ gives also $\text{SO}(10)$ enhancements, but they are not along the chain of E_n series unifications. The specially tuned form of Eq. (29) indicates a larger gauge symmetry than generic $\text{SU}(5)$, which must be $\text{SU}(5) \times \text{U}(1)_X$. We will analyze further this symmetry later.

D. Fluxes and Chiral Spectrum

The matter curves obtained in the previous section span 6 dimensional world volumes. To obtain 4 dimensional chiral spectrum, we turn-on G -flux [23]. Since the GUT group $\text{SU}(5) \times \text{U}(1)_X$ is broken by the Higgs scalar, we only need to turn-on fluxes on the spectral cover, not on the GUT sevenbranes. To keep $\sin^2 \theta_W^0 = \frac{3}{8}$, we should preserve the $\text{SO}(10)$ unification relation. Its commutant group under E_8 is $\text{SU}(4)_\perp$, and the $\text{SU}(3)_\perp$ and $\text{U}(1)_Z$

covers are identified as $C^{(a)}$ and $C^{(b)}$, respectively. Hence, we turn-on the universal fluxes only on $C^{(a)}$ and $C^{(b)}$ to preserve the $\text{SO}(10)$ structure.

First, we turn-on a line bundle \mathcal{N} on $C^{(a)}$, inducing the $\text{U}(3)$ vector bundle $V = \pi_{a*}\mathcal{N}$ on S_{GUT} :

$$\Gamma_a = \lambda \{3\sigma - \pi_a^*(\eta - 3c_1)\} + \frac{1}{3}\pi_a^*\zeta \quad (31)$$

with the projection $\pi_a : C^{(a)} \rightarrow S_{\text{GUT}}$. The trace part is $\zeta = c_1(V)$, and it is cancelled by a line bundle on $C^{(b)}$ [29],

$$\Gamma_b = -\pi_b^*\zeta \quad (32)$$

with the projection $\pi_b : C^{(b)} \rightarrow S_{\text{GUT}}$.

We have the quantization condition for \mathcal{N} [23],

$$c_1(\mathcal{N}) = \frac{1}{2} \{ -c_1(C^{(a)}) + \pi_a^*c_1 \} + \Gamma_a \in H^2(C^{(a)}, \mathbb{Z}) . \quad (33)$$

From the adjunction formula for \check{Z} , we have

$$-c_1(C^{(a)}) + \pi_a^*c_1 = \sigma + \pi_a^*(\eta - c_1) . \quad (34)$$

Thus, the quantization condition for \mathcal{N} , Eq. (33) provides the following nontrivial constraints;

$$3(\frac{1}{2} + \lambda) \in \mathbb{Z}, \quad -(\lambda - \frac{1}{2})\eta + (3\lambda - \frac{1}{2})c_1 + \frac{1}{3}\zeta \in H^2(S_{\text{GUT}}, \mathbb{Z}) . \quad (35)$$

The $C^{(d)}$ cover responsible for $\text{U}(1)_X$ is a single cover, and so we can turn-off the flux. Then the unwanted $\mathbf{10}_{-4}$ becomes vector-like, and so it can be removed from the low energy field spectrum. From now on, we will drop the symbol of pullback ‘ π^* ’ for simplicity, unless they are unclear.

The net numbers of the chiral fields are calculated using Riemann-Roch-Hirzebruch index theorem [31, 32]

$$n(R) \equiv n_R - n_{\overline{R}} = \Sigma_R \cap \Gamma|_{\sigma} \quad (36)$$

where Σ_R is the matter curve inside \check{Z} , shown in (18),(19) and (20)-(27). Specifically we

have

$$\begin{aligned}
n(\mathbf{10}_1) &= n(\overline{\mathbf{5}}_{-3}) = n(\mathbf{1}_5) \\
&= \left[(\sigma \cap (\eta - 3c_1)) \cap \left(\lambda(3\sigma_\infty - \eta) + \frac{1}{3}\zeta \right) + (\sigma \cap (-c_1)) \cap (-\zeta) \right]_\sigma \\
&= - \left(\lambda\eta - \frac{1}{3}\zeta \right) \cdot (\eta - 3c_1) + c_1 \cdot \zeta ,
\end{aligned} \tag{37}$$

$$\begin{aligned}
n(\overline{\mathbf{5}}_2) &= \left[((2\sigma + \eta) \cap (\eta - 3c_1)) \cap \left(\lambda(3\sigma_\infty - \eta) + \frac{1}{3}\zeta \right) \right]_\sigma \\
&= \left(\lambda\eta + \frac{2}{3}\zeta \right) \cdot (\eta - 3c_1) ,
\end{aligned} \tag{38}$$

$$\begin{aligned}
n(\overline{\mathbf{5}}'_2) &= \left[(\sigma \cap (\eta - 3c_1)) \cap \left(\eta(3\sigma_\infty - \eta) + \frac{1}{3}\zeta - \zeta \right) \right]_\sigma \\
&= - \left(\lambda\eta + \frac{2}{3}\zeta \right) \cdot (\eta - 3c_1) ,
\end{aligned} \tag{39}$$

$$\begin{aligned}
n(\mathbf{1}_0) &= \left[(\sigma \cap (\eta - 3c_1)) \cap \left(\eta(3\sigma_\infty - \eta) + \frac{1}{3}\zeta + \zeta \right) \right]_\sigma \\
&= - \left(\lambda\eta - \frac{4}{3}\zeta \right) \cdot (\eta - 3c_1) .
\end{aligned} \tag{40}$$

Here the intersection is done in the \check{Z} space, and the dot product is done on S_{GUT} . Thus, the existence of three families of the SM matter and only one pair of the vector-like electroweak Higgs fields require $-(\lambda\eta - \frac{1}{3}\zeta) \cdot (\eta - 3c_1) + c_1 \cdot \zeta = 3$ and $-(\lambda\eta + \frac{2}{3}\zeta) \cdot (\eta - 3c_1) = 1$. To kill the unwanted superpotential terms, $\mathbf{10}_1^{(\prime)} \mathbf{10}_1^{(\prime)} \mathbf{10}_1^{(\prime)} \overline{\mathbf{5}}_{-3}^{(\prime)}$ and $\mathbf{10}_H \mathbf{10}_1^{(\prime)} \mathbf{10}_1^{(\prime)} \overline{\mathbf{5}}_{-3}^{(\prime)}$, as mentioned above, the *matter* fields associated with $t_4 \rightarrow 0$ i.e. $\mathbf{10}'_1$, $\overline{\mathbf{5}}'_{-3}$, and $\mathbf{1}'_5$ should be absent at low energies. Hence, we take

$$\lambda\eta \cdot (\eta - 3c_1) = -\frac{7}{3} , \quad \eta \cdot \zeta = 2 , \quad \text{and} \quad c_1 \cdot \zeta = 0 \tag{41}$$

Moreover, the absence of a flux on $C^{(d)}$ leaves the exotic field $\mathbf{10}_{-4}$ vector-like. The zero modes of the chiral field spectrum are summarized in Table II.

For constructing a local model, a necessary condition is that the four cycle S_{GUT} is a del Pezzo surface dP_n [10]: S_{GUT} should be shrinkable inside the ambient space. In global model it is not necessary but del Pezzo surface is easy to realize as a projective variety. The first constraint in Eq. (35) is easily fulfilled by taking $\lambda = \frac{1}{6}$. Then the second constraint in Eq. (35) implies

$$\frac{1}{3}(\eta + \zeta) \in H_2(S_{\text{GUT}}, \mathbb{Z}) \tag{42}$$

for $\lambda = \frac{1}{6}$. We can find η and ζ satisfying Eqs. (41), e.g. just if $S_{\text{GUT}} = dP_2$, namely, the canonical class is given by $-K_S = c_1 = 3H - E_1 - E_2$:

$$\eta = 2H , \quad \zeta = H - 3E_1 , \tag{43}$$

where H and E_i ($i = 1, 2$) denote the hyperplane divisor and exceptional divisors, respectively, satisfying

$$H \cdot H = 1, \quad E_i \cdot E_j = -\delta_{ij}, \quad \text{and} \quad H \cdot E_i = 0. \quad (44)$$

The global embedding is easily done by borrowing the dP_2 construction in [33].

IV. ABELIAN SYMMETRY

A. $U(1)_X$ as Gauge Symmetry

There is an issue concerning $U(1)$ gauge group [30, 34, 35]. Since there is only one Cartan subalgebra, we cannot identify it geometrically. The Cartan subalgebra are obtained by reducing the three-form field along two-cycles, and their field strength satisfies

$$G = \sum F \wedge \omega, \quad \omega \in H^2(CY_4, \mathbb{Z}), \quad (45)$$

using collective notation, where G is four-form field strength analogous to one in M-theory and two forms ω are not in the three-base or the elliptic fiber. If we turn-on a line bundle on this cover, then it potentially makes the corresponding gauge boson massive by the Stückelberg mechanism. From the interaction involving G we have the induced action [13],

$$\int_{\mathbb{R}^{1,3}} F_X \wedge c_2^{(i)} \text{tr} X^2 \int_S c_1(L_X) \wedge \iota^* \omega_i. \quad (46)$$

Note that the contribution from $U(1)_X$ charges is proportional to $\text{tr} X^2$. Even if there is no 4 dimensional gauge and gravitational anomalies proportional to $\text{tr} X$ or $\text{tr} X^3$, still there is a room for massive gauge bosons. In our situation we do not turn on flux along the $U(1)_X$, $L_X = 0$, there the gauge boson is massless from (46).

We can check that there is no $U(1)_X$ gauge and gravitational anomalies

$$\sum_R n(R) \dim(R) X^3 = 0, \quad \sum_R n(R) \ell(R) X = 0, \quad \sum_R n(R) \dim(R) X = 0,$$

where $n(R)$ is the net number of chiral minus antichiral generations for R . $\ell(R)$ denotes the Dynkin index defined as

$$\text{tr}_R T^a T^b = \ell(R) \delta^{ab}, \quad (47)$$

for the generators T^a of a Lie group G . Since there is no missing charged matter, it seems that this $U(1)_X$ can be fully understood in local description. A supporting argument is, in the case where $U(1)_X$ is protected by a larger nonabelian gauge group (e.g. as in Ref. [28]) e.g. $SO(10)$ as in our case, we can arbitrarily shrink the two-cycle where $U(1)_X$ is local enough, being caught around S_{GUT} . There is no new monodromy mixed with the cycles outside E_8 . In some literatures, the condition for six dimensional anomaly cancellation conditions Eq. (16) were not met, and so the identification of $SU(5)$ singlets carrying $U(1)_X$ charges was failed.

B. U(1) Normalization

We explain how the normalization of $U(1)_X$ is determined when flipped $SU(5)$ is embedded in $SO(10)$. Formerly, our definition of $U(1)_X$ in Eq. (9) does not rely on the $SO(10)$, but embedded in the $SU(5)_\perp$, which is the commutant group of the GUT $SU(5)$ in E_8 . We will see how they are related.

In dealing with normalization, the Dynkin index defined in Eq. (47) is useful. Once we fix $\ell(R)$ for one kind of representation R , it fixes the normalization of all the generators of the group G . For example, the complex conjugate representation \bar{R} of R has the relation $\ell(\bar{R}) = \ell(R)$.

Considering a subgroup H of G and the commutant group Γ , there is a property

$$R \rightarrow \sum (R_H, R_\Gamma), \quad \ell(R) = \sum \ell(R_H) \dim(R_\Gamma). \quad (48)$$

As an example, consider $SU(n)$ and its subgroup $SU(2)$. Fixing $\ell(\mathbf{2}) = \frac{1}{2}$ for $SU(2)$, also fixes $\ell(\mathbf{n}) = \frac{1}{2}$ for the fundamental representation of $SU(n)$. It is easily shown by relation Eq. (48) and using the fact that the singlet is neutral under the group $\ell(\mathbf{1}) = 0$. The relation Eq. (48) is unique so that the converse also holds. Starting from any group G , we can show the same relation to its $SU(2)$ subgroup in *any direction*. An important consequence is that

$$\text{the generators of any SU-type subgroup of } G \text{ have the same normalization} \quad (49)$$

in the fundamental representation.

Now, consider $SU(5) \times U(1)_X$ subgroup of $SO(10)$. This embedding is easily understood by conventional ‘complexification.’ Since $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$ in $SU(5)$ are embedded in a vector representation $\mathbf{10}$ of $SO(10)$, the *same* $U(1)_X$ generator has two *different representations*, $T_X^{\mathbf{10}}$ with respect to $\mathbf{10}$ of $SO(10)$ and $T_X^{\mathbf{5}}$ to $\mathbf{5}$ of $SU(5)$, for example,

$$T_X^{\mathbf{10}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes iT_X^{\mathbf{5}'} \xrightarrow{\text{diagonalization}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes T_X^{\mathbf{5}}. \quad (50)$$

Explicitly, $T_X^{\mathbf{5}} = \text{diag}(-2, -2, -2, -2, -2)$. This shows, once we fix the normalization $\ell(\bar{\mathbf{5}}) = \frac{1}{2}$ of $SU(5)$, $SO(10)$ vector has the normalization $\ell(\mathbf{10}) = 1$, as it should be from Eq. (48). In other words,

$$|(-2, -2, -2, -2, -2; 2, 2, 2, 2, 2)|^2 / \ell(\mathbf{10}) = |(-2, -2, -2, -2, -2)|^2 / \ell(\mathbf{5}) = 40, \quad (51)$$

in the $SU(5)$ basis. It implies the normalized generator is $\frac{1}{\sqrt{40}} T_X^{\mathbf{5}}$.

In our case, $U(1)_X$ of flipped $SU(5)$ is embedded in the $SU(5)_\perp$, which is the commutant group of the GUT $SU(5)$ in E_8 , as shown in Eq. (9). So at first sight its generator X seems not be related to the previous $SO(10)$. However $U(1)_X$ is the common intersection between $SO(10)$ and $SU(5)_\perp$, so we observe that X and $T_X^{\mathbf{5}}$ are the *same* generators with merely different representations. This is of course understood as being E_8 generator. Here we check

explicitly their normalizations are the same. A particular case of Eq. (49) is that, fixing $\ell(\bar{\mathbf{5}}) = \frac{1}{2}$ for the $SU(5)$, it should be that $\ell(\bar{\mathbf{5}}_\perp) = \frac{1}{2}$ for the other $SU(5)_\perp$. As a generator of $SU(5)_\perp$, X should be replaced with the one with normalization $\text{tr}\tilde{X}^2 = \frac{1}{2}$. Indeed,

$$|(-2, -2, -2, -2, -2)|^2/\ell(\mathbf{5}) = |(1, 1, 1, 1, -4)|^2/\ell(\mathbf{5}_\perp) = 40. \quad (52)$$

Since we independently identified the generators, Eq. (52) gives a nontrivial check that

$$\frac{1}{\sqrt{40}}T_X^{\mathbf{5}}, \quad \frac{1}{\sqrt{40}}X. \quad (53)$$

should be different representations of a *single* generator of E_8 .³

When flipped $SU(5)$ is broken to the SM gauge group, the gauge coupling of $U(1)_Y$, g_Y in the SM becomes related to g_5 and g_X of flipped $SU(5)$. We recollect the gauge kinetic terms for $SU(5) \times U(1)_X$ from that of $SO(10)$. Using the relation $Y = \frac{1}{5}(T_5 + X)$ as in Eq. (1) and the normalization in Eq. (52), we extract the coupling relation,

$$-\frac{1}{4g_{SO(10)}^2}\text{tr}_{\mathbf{10}}F^2 \rightarrow -\frac{1}{2g_5^2} \cdot \frac{1}{5^2}\text{tr}_{\mathbf{5}}(F_{T_5}^2 + F_X^2) = -\frac{1}{4g_Y^2}F_Y^2,$$

so that

$$\frac{1}{g_Y^2} = \frac{1}{5^2} \cdot \frac{5}{3} \cdot \frac{1}{g_5^2} + \frac{1}{5^2} \cdot 40 \cdot \frac{1}{g_X^2}, \quad (54)$$

from which we understand $g_X = g_5$ at the GUT scale. We have $g_2 = g_5$, since $SU(2)_L$ in the SM gauge group comes purely from the $SU(5)$ part of flipped $SU(5)$, thus the bare weak mixing angle at the GUT scale is

$$\sin^2\theta_W^0 \equiv \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{1}{\frac{g_5^2}{g_Y^2} + 1} = \frac{3}{8}. \quad (55)$$

³ It also follows that the normalized one for Eq. (13) should be $\text{tr}\tilde{Z}^2 = \frac{1}{2}$ from the embedding $SU(3) \times U(1)_Z \subset SU(4)$. Its commutant in E_8 is $SO(10)$, so $U(1)_Z$ is the common intersection of the $SO(10)$ and the $SU(4)$. Fixing the normalization $\ell(\mathbf{4}) = \frac{1}{2}$ means also fixing $\ell(\mathbf{27}) = 3$. It is done, for example, by considering the chain $\ell(\mathbf{27}) = \ell(\mathbf{16}_1) + \ell(\mathbf{10}_{-2}) + \ell(\mathbf{1}_4) = 2 + 1 + 0$. Therefore we fix the normalization of $U(1)_Z$ generator inside E_6 , with respect to the minimal representation $\mathbf{27}$,

$$|\underbrace{(1, 1, \dots, 1)}_{16}, \underbrace{(-2, -2, \dots, -2)}_{10}, 4|^2/\ell(\mathbf{27}) = |(1, 1, 1, -3)|^2/\ell(\mathbf{4}) = 24.$$

Here the bracing numbers indicate the number of repeated entries. Also, identifying the SM group as $E_3 \times U(1)_Y$, the commutant to $SU(5) \times U(1)_Y$ in E_8 , from the embedding to $SU(6)$, we have a similar relation

$$|(-2, -2, -2, 3, 3)|^2/\ell(\mathbf{5}) = |(1, 1, 1, 1, -5)|^2/\ell(\mathbf{6}) = 60.$$

Why does flipped SU(5) yield the same relation as SU(5)_{GG}? If $U(1)_X$ is embedded in a simple group of SO(10), there is a single coupling. The main difference between GG SU(5) and flipped SU(5) comes from the definition of hypercharges

$$Y_{\text{GG}} = \sqrt{\frac{3}{5}} \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{-1}{2}, \frac{-1}{2}), \quad Y_{\text{F-SU(5)}} = \sqrt{\frac{3}{5}} \text{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}),$$

in the fundamental representation. They just differ by some signs of hypercharges while SU(3)_C × SU(2)_L direction intact. So, from the relation Eq. (55), the same g_Y normalization gives the same weak mixing angle. The new hypercharge combination is possible because flipped SU(5) has an extra component of the $U(1)_X$, as in (9). It turns out that this is the only possible new combination inside SO(10).

The coupling unification and the weak mixing angle relation is the same if the symmetry breaking mechanism down to MSSM does not change gauge coupling. It includes scalar Higgses and Wilson lines. Such mechanism is sometimes associated with the symmetry of internal manifold, so heterotic string compactification with a background gauge bundle, or F-theory with a spectral cover with the *hypercharge untouched* gives the same unification and weak mixing angle. In F-theory, there is another source of gauge symmetry breaking: G -flux along hypercharge direction gives a correction to gauge kinetic function thus gauge coupling. So the relation changes slightly [12, 16, 17]. The simplest GUT that does not require hypercharge flux is flipped SU(5), so still the relation is preserved.

V. LOW ENERGY EFFECTIVE THEORY

In our F-theory model we obtain 3 *net* families of $\mathbf{10}_1$ s, 5 of $\mathbf{1}_0$ s, and one net pair of $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$. In particular, $\mathbf{10}_1$ and $\bar{\mathbf{10}}_{-1}$ matter representations belong to cohomology $H^0(\Sigma_{\mathbf{10}}, K_{\Sigma_{\mathbf{10}}}^{1/2} \otimes V)$ and $H^0(\Sigma_{\bar{\mathbf{10}}}, K_{\Sigma_{\bar{\mathbf{10}}}}^{1/2} \otimes V^*)$, respectively. At present it is not possible to calculate the individual Euler numbers for them, we suppose $4 \times \mathbf{10}_1$ and $1 \times \bar{\mathbf{10}}_{-1}$, and regard $\bar{\mathbf{10}}_{-1}$ and one of $\mathbf{10}_1$ as $\bar{\mathbf{10}}_H$ and $\mathbf{10}_H$ breaking flipped SU(5), respectively. On the other hand, we assume that the absolute number of $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$ is 1.

The VEV distinguishes $\mathbf{10}_H$ from $\mathbf{10}_i$. Since $\mathbf{10}_H$ and $\bar{\mathbf{10}}_H$ have the exactly opposite gauge quantum numbers, the superpotential admits the following terms;

$$W \supset M_G \mathbf{10}_H \bar{\mathbf{10}}_H + \frac{1}{M_G} (\mathbf{10}_H \bar{\mathbf{10}}_H)^2 + \dots, \quad (56)$$

where we assume that the fundamental scale is of the GUT scale as in the heterotic M theory. From these terms in the superpotential and the D-term potential, $\mathbf{10}_H$ and $\bar{\mathbf{10}}_H$ can develop a VEV at a *SUSY vacuum*, $\langle \mathbf{10}_H \rangle = \langle \bar{\mathbf{10}}_H \rangle \sim \mathcal{O}(M_G)$, satisfying $\partial W / \partial \mathbf{10}_H = \partial W / \partial \bar{\mathbf{10}}_H = 0$ and $\langle \mathbf{10}_H \rangle = \langle \bar{\mathbf{10}}_H \rangle^*$. Just based on the symmetries discussed above, the

expected low energy superpotential (upto dimension 5) is given by

$$W_{\text{eff}} = (\mathbf{10}_H \mathbf{10}_H \mathbf{5}_h + \mathbf{10}_H \mathbf{10}_i \mathbf{5}_h) + \mathbf{10}_i \mathbf{10}_j \mathbf{5}_h + \frac{1}{M_G} \overline{\mathbf{10}}_H \overline{\mathbf{10}}_H \overline{\mathbf{5}}_h S' \quad (57)$$

$$+ (\mathbf{10}_H \overline{\mathbf{5}}_i \overline{\mathbf{5}}_h + S \mathbf{5}_h \overline{\mathbf{5}}_h) + \mathbf{10}_i \overline{\mathbf{5}}_j \overline{\mathbf{5}}_h + \mathbf{1}_i \overline{\mathbf{5}}_j \mathbf{5}_h + \frac{1}{M_G} \overline{\mathbf{10}}_H \overline{\mathbf{10}}_H \mathbf{10}_i \mathbf{10}_j ,$$

where we drop the dimensionless coupling constants for simplicity. As explained earlier, the unwanted terms Eqs. (4), (5), and (6) are absent. S and S' denote the different linear combinations of five $\mathbf{1}_0$ s associated with the matter curve of $\prod_i(t_1 - t_4) \rightarrow 0$.

Since $\mathbf{10}_i$ and $\mathbf{10}_H$ have the same charges in this model, both $\mathbf{10}_H \mathbf{10}_H \mathbf{5}_h + \mathbf{10}_H \mathbf{10}_i \mathbf{5}_h$ are allowed as seen in Eq. (57). It gives

$$\langle \nu_H^c \rangle (d_H^c + d_1^c + d_2^c + d_3^c) D . \quad (58)$$

Hence, the mode $(d_H^c + d_1^c + d_2^c + d_3^c)$ and also D become heavy, whereas the other 3 modes orthogonal to $(d_H^c + d_1^c + d_2^c + d_3^c)$ can be regarded as the physical d-type quarks. This mixing could suppress the d-type quark's Yukawa couplings in $\mathbf{10}_i \mathbf{10}_j \mathbf{5}_h$. The mixing between d_H^c and d_i^c might be helpful for explaining $m_b/m_t \sim \mathcal{O}(10^{-2})$.

Similarly, $\mathbf{10}_H \overline{\mathbf{5}}_i \overline{\mathbf{5}}_h$ and $S \mathbf{5}_h \overline{\mathbf{5}}_h$ make l_i and h_d mixed:

$$\{\langle \nu_H^c \rangle (l_1 + l_2 + l_3) + S h_d\} h_u = \mu h'_d h_u , \quad (59)$$

where h'_d defines the physical d-type Higgs, and μ is given by

$$\mu = \sqrt{\langle \nu_H^c \rangle^2 + \langle S \rangle^2} . \quad (60)$$

Note that ν_H^c and S are complex fields. Due to the mixing between l_i and h_d , some R-parity violating terms $q_i l_j d_j^c$ and $l_i l_j e_k^c$ in the superpotential are induced, but the other term $u_i^c d_j^c d_k^c$ is not. $q_i l_j d_j^c$ and $l_i l_j e_k^c$ violate lepton numbers, but still preserve the baryon number. Since the dimension 4 proton decay processes are associated with both $q_i l_j d_j^c$ and $u_i^c d_j^c d_k^c$, still the proton can be stable enough.

The smallest one among the upper bounds for the dimension less lepton number violating couplings is around 10^{-6} [36]. It is a similar size of the (R-parity preserving) electron's Yukawa couplings. In this model, the accidental global symmetry found at low energies is Z_3 , under which the MSSM superfields carry the charges; $q(0), u^c(-1), d^c(1), l(-1), \nu^c(0), e^c(2), h_u(1), h_d(-1)$ [37].

The R-parity violating terms arise since both $\mathbf{10}_i$ and the Higgs $\mathbf{10}_H$ are parameterized by a single component t_i . Conventionally they are distinguished by imposing R -parity in field theoretic GUT: $(-)$ for the matter fields and $(+)$ for the Higgs fields. However, there is no known way to embed it to a continuous symmetry [See also [15]]. Maybe we keep it as an accidental symmetry up to a certain order of perturbation that is suppressed enough [38]. In our type factorization, there appears another representation $\mathbf{10}'_1$, with which one

may attempt to identify as the Higgs. However this leads to proton decay operators Eq. (4). Hence, one may consider a further factorizing the spectral cover type. However, we will not pursue this possibility for simplicity of the model, just assuming the relatively small lepton number violating couplings.

As noticed above, S is composed of the five $\mathbf{1}_0$ s, and the VEV of S , i.e. all of $\mathbf{1}_0$ s could remain undetermined down to low energies. In this case, $\langle S \rangle$ (as well as $\langle h_u \rangle$ and $\langle h_d \rangle$) would be eventually fixed by including TeV scale SUSY breaking “soft terms” in the Lagrangian such that μ of Eq. (60) becomes of TeV scale. It implies that $\langle S' \rangle$ in Eq. (57) should be of order $\langle \mathbf{10}_H \rangle$ ($\sim M_G$), because S and S' are the different linear combinations of five $\mathbf{1}_0$ s. $\langle S' \rangle$ of order M_G induces the second term of Eq. (3).

In this mechanism, a modulus, which is given by a linear combination of $\mathbf{1}_0$ s, plays an essential role. However, it would give rise to the cosmological “moduli problem.” Since its VEV is around the GUT scale but its mass is just of TeV scale, its decay is not efficient and its oscillation around the true minimum of the scalar potential is hard to be terminated. This problem could be resolved by considering the second inflation around TeV scale temperature (“thermal inflation”) [39]. In this paper, we don’t discuss this issue in details.

VI. CONCLUSIONS

In this paper, we have pointed out that the normalization of the $U(1)_X$ charges in flipped $SU(5)$ models based on F-theory can be determined such that $\sin^2 \theta_W^0 = \frac{3}{8}$. It is because $U(1)_X$ is embedded in a simple structure group $SU(5)_\perp \subset E_8$. To avoid the dimension 4 and 5 proton decay, the structure group should split to $SU(3)_\perp$ or smaller group factors, and extra heavy vector-like pairs $\{\mathbf{5}_{-2}, \bar{\mathbf{5}}_2\}$ should be absent. We have proposed a simple but phenomenologically viable flipped $SU(5)$ model based on F-theory.

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